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# Suppression of thermally activated escape by heating

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The problem of thermally activated escape over a potential barrier is solved by means of path integrals for one-dimensional reaction dynamics with very general time dependences. For a suitably chosen but still quite simple static potential landscape, the net escape rate may be substantially reduced by temporally increasing the temperature above its unperturbed constant level.

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Thermally activated escape over potential barriers is of relevance in a large variety of physical, chemical, and biological contexts [1]. In the most common case, a potential barrier $\Delta U$ much larger than the thermal energy $kT$ yields an escape rate exponentially small in $\Delta U/kT$. A first major generalization, of importance for conceptual reasons as well as due to numerous applications, are periodically modulated potentials [2–6], resulting in a renormalization of $\Delta U$ which depends in a very complicated manner on the details of the model. In all these cases the rate is thus a very rapidly increasing function of the temperature $T$. In our present work, the main focus is on the complementary case of a static potential and a time-dependent temperature [6,7]. In particular, we will demonstrate that in a suitably chosen but still fairly simple and generic potential landscape, the escape rate of the unperturbed system at constant temperature may *decrease* upon temporally *increasing* the temperature. In view of the above-mentioned results for constant temperature, this is a quite unexpected and counterintuitive result. Indeed, given that thermal noise is indispensable to escape, one would expect that an "extra dose" of noise should always enhance escape. Somewhat reminiscent previous findings always concern quite different types of systems: dissipative quantum tunneling in the deep cold [8], activationless electron transfer [9], complex protein dynamics near the solvent glass transition [10], models without a barrier against deterministic escape [11], or nondynamical systems [12].

As our main tool, we put forward a path integral approach, which unifies and extends several related approximations [2–6,13,14]. Briefly, in different parameter regimes of the temporal modulations, the most relevant escape paths are of quite different character. Therefore, each regime was so far treated separately and the crossover omitted. Here, all potentially relevant paths are represented in terms of a suitable one-dimensional (1D) parametrization and are kept till the final rate formula via an integral over all of them.

We consider the overdamped 1D Langevin equation

$$\eta(t)\dot{x}(t) = -U'[x(t),t] + \sqrt{2\eta(t)kT(t)}\xi(t), \tag{1}$$

with time-dependent friction $\eta(t)>0$, temperature $T(t)>0$, and potential $U(x,t)$. Dot and prime indicate temporal and spatial derivatives, $k$ is Boltzmann's constant, and thermal fluctuations are modeled as usual [1] by $\delta$-correlated Gaussian noise $\xi(t)$. For $T\to 0$, the deterministic dynamics is required to exhibit exactly one stable orbit (attractor) $x_s(t)$ and one unstable orbit (repeller) $x_u(t)>x_s(t)$. Our main interest concerns the noise-induced transitions of $x(t)$ across $x_u(t)$ for small but finite temperatures $T(t)$ quantified by the rate $\Gamma(t) := -\dot{n}(t)$ at which the probability $n(t)$ that $x(t)\le x_u(t)$ changes in time.

To avoid unnecessary complications, we focus on initial conditions $x(t_0)=x_s(t_0)$, and we require the existence of $D := \lim_{t\to\infty}\int_0^t \frac{d\tau}{t} b(\tau)$ with $b(t) := kT(t)/\eta(t)$. Next, we divide Eq. (1) by $\eta(t)$ and employ transformed times $\tilde{t}(t) := \int_0^t d\tau\, b(\tau)/D$, positions $\tilde{x}(\tilde{t}) := x[t(\tilde{t})]$, and forces $\tilde{F}(x,\tilde{t}) := -DU'[x,t(\tilde{t})]/kT[t(\tilde{t})]$, yielding, after dropping again the tildes,

$$\dot{x}(t) = F[x(t),t] + \sqrt{2D}\xi(t). \tag{2}$$

In the general formalism, we will work with Eq. (2), while specific examples will refer to Eq. (1). The corresponding (back)transformation of the rates $\Gamma(t)=b(t)\tilde{\Gamma}[\tilde{t}(t)]/D$ readily follows from the obvious transformation of the probabilities $\tilde{n}(\tilde{t})=n[t(\tilde{t})]$.

We first recall some basics, previously derived and discussed in detail in Ref. [5]: for any given initial condition $x(t_0)=x_0$, the probability density to find the stochastic process (2) at any "final" time $t_f>t_0$ at the position $x_f$ can be represented as path integral

$$\rho(x_f,t_f|x_0,t_0) = \int_{x(t_0)=x_0}^{x(t_f)=x_f} \mathcal{D}x(t)\, e^{-S[x(t)]/D}, \tag{3}$$

with action $S[x(t)] := \int_{t_0}^{t_f} dt\{\dot{x}(t)-F[x(t),t]\}^2/4$. Once this formal integral is evaluated, the rate follows as

$$\Gamma(t) = -D\,\partial\,\rho(x_u(t),t|x_s(t_0),t_0)/\partial x_u(t). \tag{4}$$

For small $D$, the integral (3) is dominated by the path $q(t)$ which minimizes the action $S[x(t)]$ and thus satisfies the Euler-Lagrange equation

$$\dot{p}(t) = -p(t)F'(q(t),t), \quad p(t) := \dot{q}(t) - F(q(t),t), \tag{5}$$

with boundary conditions $q(t_0)=x_0$ and $q(t_f)=x_f$. Accounting for all paths $x(t)$ "close" to $q(t)$ by means of a functional saddle-point approximation in Eq. (3) yields

$$\rho(x_f,t_f|x_0,t_0) = [4\pi DQ(t_f)]^{-1/2} e^{-S[q(t)]/D}, \tag{6}$$

where $Q(t)$ satisfies $Q(t_0)=0$, $\dot{Q}(t_0)=1$, and

 

$$\ddot{Q}(t) = \frac{d}{dt}\{2Q(t)F'[q(t),t]\} - Q(t)p(t)F''[q(t),t]. \quad (7)$$

Equations (4) and (6) yield an approximation to the rate $\Gamma(t)$, which in principle becomes asymptotically exact as $D \to 0$ for any given $t > t_0$ with a unique absolute minimum of $S[x(t)]$, which generically is the case. But under many circumstances of foremost interest (e.g., relatively large $t - t_0$) even fairly small $D$ are still far from this asymptotic regime, i.e., the saddle-point approximation [Eq. (6)] does not properly account for all relevant paths in Eq. (3): Basically, a typical escape path $x(t)$ spends almost all its time near $x_s(t)$, then crosses over into the vicinity of $x_u(t)$, and remains there for the rest of its time. Any other behavior would yield a much larger action $S[x(t)]$ and thus is negligible in Eq. (3). However, rather different crossover "time windows" may still lead to almost equal $S[x(t)]$, and a simple saddle-point approximation is unable to properly account for such quite remote regions in path space. In some cases, there may exist further local minima of $S[x(t)]$ and additional saddle-point approximations around each of them may save the case [3,5]. The remaining problem is to keep track of all relevant minima and not to double count their neighborhoods if they get too close in path space. In other cases, e.g., for $t$ independent $\eta$, $T$, and $U$ in Eq. (1), there is a continuous "soft direction" in path space, invalidating plain saddle-point methods altogether [13].

To overcome these problems we impose on top of the boundary conditions $x(t_0) = x_s(t_0)$ and $x(t_f) = x_u(t_f)$ the extra condition that $x(t)$ arrives at some intermediate point $x_i$ at a given time $t_i$, and in the end integrate over all $t_i \in [t_0, t_f]$ [15]. The pertinent formal relation, satisfied by the conditional probability density [Eq. (3)], is

$$\rho(x_f, t_f | x_0, t_0) = \int_{t_0}^{t_f} dt_i \, \rho(x_f, t_f | x_i, t_i) \Psi_{x_i}(t_i | x_0, t_0), \quad (8)$$

where $\Psi_x(t | x_0, t_0)$ denotes the first passage time density across $x$, given $x(t_0) = x_0$. For simplicity only, we assume from now on that $x_i$ is located well in between $x_s(t)$ and $x_u(t)$ and is $t$ independent. Then, all non-negligible paths in Eq. (3) starting from $x(t_i) = x_i$ must immediately cross over to $x_u(t)$ and thus admit for $\rho(x_f, t_f | x_i, t_i)$ in Eq. (8) a saddle-point approximation [Eq. (6)] free of all the above-mentioned problems. Focusing on $x_0 = x_s(t_0)$ according to Eq. (4), an analogous approximation (6) holds for $\rho(x_i, t_i | x_0, t_0)$ since all relevant paths in Eq. (3) now may leave the vicinity of $x_s(t)$ only in the very end. By definition, $\rho(x_i, t_i | x_0, t_0) dx_i$ is the probability that $x(t)$ from Eq. (2) is encountered within $[x_i, x_i + dx_i]$ at time $t_i$, given $x(t_0) = x_0 = x_s(t_0)$. Most such $x(t)$ closely resemble the most probable path $q(t)$ connecting $q(t_0) = x_0$ with $q(t_i) = x_i$. On the other hand $\Psi_{x_i}(t_i | x_0, t_0) dt_i$ is the probability that $x(t)$ crosses $x_i$ for the first time during $t \in [t_i, t_i + dt_i]$. It seems reasonable to guess that most such $x(t)$ once again closely resemble $q(t)$. Hence, $\rho(x_i, t_i | x_0, t_0) dx_i$ will essentially account for the same "events" as $\Psi_{x_i}(t_i | x_0, t_0) dt_i$ provided we relate the considered intervals $dx_i$ and $dt_i$ via $dx_i = \dot{q}(t_i) dt_i$. Up to finite-$D$ corrections we thus obtain

$$\Psi_{x_i}(t_i | x_0, t_0) = \dot{q}(t_i) \rho(x_i, t_i | x_0, t_0), \quad (9)$$

where $q(t)$ satisfies $q(t_0) = x_0 = x_s(t_0)$, $q(t_i) = x_i$, and Eq. (5). More rigorously, our key relation (9) follows by adapting Ref. [16] to evaluate the derivative by $x_f$ of Eq. (8) in the limit $x_f \to x_i$. Details will be given elsewhere.

The evaluation of the escape rate [Eq. (4)] by means of Eqs. (6), (8), and (9) is the first main result of our present work. Similarly as in [3–6], closed analytical solutions of the concomitant differential Eqs. (5) and (7) are only possible for special $F(x,t)$. To this end, we focus on piecewise parabolic potentials $U(x,t)$ in Eq. (1), corresponding to piecewise linear force fields in Eq. (2) of the form

$$F(x \le 0, t) = \lambda_s(t)[x - y_s(t)] + f(t),$$

$$F(x > 0, t) = \lambda_u(t)[x - y_u(t)] + f(t), \quad (10)$$

with $\lambda_s(t) y_s(t) = \lambda_u(t) y_u(t)$ (continuity at $x = 0$). Further, the existence of stable and unstable orbits with $x_s(t) < x_i$ and $x_u(t) > x_i$ is required, in particular $y_s(t), \lambda_s(t) < 0$, $y_u(t), \lambda_u(t) > 0$. For the natural choice $x_i = 0$, a straightforward but somewhat tedious calculation [5,6] then yields for the rate [Eq. (4)] the result

$$\Gamma(t) = \int_{t_0}^{t} d\tau \frac{Z(t, \tau, t_0)}{D} e^{-\Phi(t, \tau, t_0)/D}, \quad (11)$$

$$\Phi(t, \tau, t_0) := \frac{x_u^2(\tau)}{4 I_u(\tau, t)} + \frac{x_s^2(\tau)}{4 I_s(\tau, t_0)}, \quad (12)$$

$$Z(t, \tau, t_0) := \frac{[Y(\tau, t_0) - x_s(\tau)] x_u(\tau)}{8\pi [I_u(\tau, t) I_s(\tau, t_0)]^{3/2}} e^{\Lambda_u(\tau, t)}, \quad (13)$$

$$Y(\tau, t_0) := I_s(\tau, t_0)[f(\tau) - y_s(\tau)\lambda_s(\tau)], \quad (14)$$

$$\Lambda_{s,u}(t, \tilde{t}) := 2 \int_{\tilde{t}}^{t} d\tau \, \lambda_{s,u}(\tau), \quad (15)$$

$$I_{s,u}(t, \tilde{t}) := \left| \int_{\tilde{t}}^{t} d\tau \, e^{\Lambda_{s,u}(t, \tau)} \right|. \quad (16)$$

We have verified that previous findings for time-periodic [3,5,6] and time-independent systems [Eq. (1)] [13] are recovered as special cases. Those from [4] are formally similar but contain quantities [called $\mathcal{E}$ and $s(\phi)$] which are not explicitly available in general.

As a first example we consider the dynamics [Eq. (1)] with constant friction $\eta(t) \equiv 1$, a temperature pulse $T(t)$ according to Fig. 1(a), and a static piecewise parabolic potential $U(x)$, see Fig. 1(b). Already for the moderately small temperatures from Fig. 1(c), the accuracy of the analytical approximation [Eq. (11)] is quite good. We found that it quickly improves even further upon decreasing temperatures. After initial transients [omitted in Fig. 1(c)], Kramers rate is recovered until the temperature pulse sets in at $t = 0$. Then, the rate rapidly increases and approaches the Kramers rate corresponding to $T_0 + \Delta T$, provided the pulse lasts suffi-

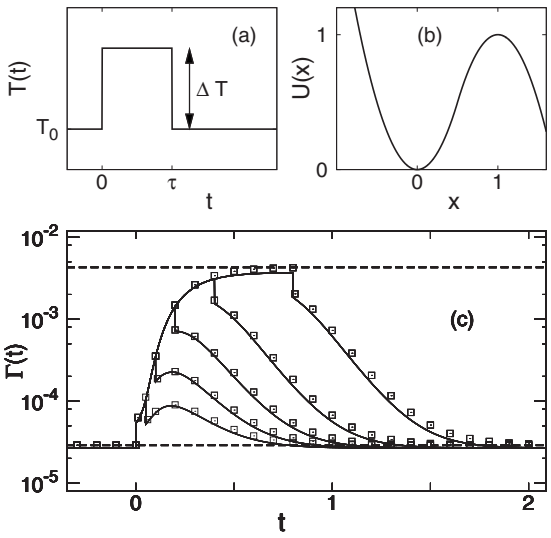


FIG. 1. (a) Temperature pulse with parameters $T_0$, $\Delta T$, and $\tau$. (b) Piecewise parabolic potential with a well at $x_s=0$, a barrier at $x_u=1$, curvatures $U''(x_s)=4$, $U''(x_u)=-4$, and barrier height $\Delta U=U(x_u)-U(x_s)=1$. (c) Time dependent escape rates for $kT_0=k\Delta T=0.1$, $\eta(t)\equiv 1$, and $\tau=0.05$, $\tau=0.1$, $\tau=0.2$, $\tau=0.4$, $\tau=0.8$ (bottom up). Squares: precise numerical solutions of Eq. (1) with seed $x(t_0=-5)=x_s$. Solid lines: analytical approximation (11) for the equivalent transformed dynamics [Eqs. (2) and (10)]. Dashed lines: Kramers rates [1] for $T=T_0$ (bottom) and $T=T_0+\Delta T$ (top).

ciently long. Finally, an analogous relaxation back to the original Kramers rate follows. Discontinuities of $T(t)$ entail jumps of $\Gamma(t)$. While the initial transients are well understood [14], to the best of our knowledge no previously existing analytical approximation would be able to faithfully describe the "perturbed and interfering transients" for largely arbitrary pulses and pulse sequences.

Next, we consider Eq. (1) with a periodically pulsating temperature $T(t)$ and a piecewise parabolic potential $U(x)$ exhibiting two barriers and two wells, see Fig. 2. Transitions from $x_1^s$ to $x_2^s$ are described by the rate $\Gamma_{1\to 2}(t)$, those from $x_2^s$ to $x_1^s$ by $\Gamma_{2\to 1}(t)$, and those from $x_2^s$ toward $x=\infty$ by $\Gamma_{2\to\infty}(t)$. After suitable time and space transformations [cf. Eq. (2)], each rate can be approximated according to Eq. (11). Since they are small (transitions are rare), it is sufficient—as far as the populations $n_1(t)$ and $n_2(t)$ of the two wells are concerned—to consider their averages over one period $\mathcal{T}$, denoted by $\bar{\Gamma}_{1\to 2}$, $\bar{\Gamma}_{2\to 1}$, and $\bar{\Gamma}_{2\to\infty}$. Then, the populations $\vec{n}(t):=(n_1(t),n_2(t))$ are governed by the master equation [1] $\dot{\vec{n}}(t)=-M\vec{n}(t)$ with matrix elements $M_{11}=-M_{21}=\bar{\Gamma}_{1\to 2}$, $M_{12}=-\bar{\Gamma}_{2\to 1}$, and $M_{22}=\bar{\Gamma}_{2\to 1}+\bar{\Gamma}_{2\to\infty}$. The smallest eigenvalue of $M$ is denoted by $\bar{\Gamma}$ and represents the ultimate rate of escape toward $\infty$ after initial relaxation processes, governed by the other eigenvalue of $M$, have died out. Since the two eigenvalues differ by a huge, Boltzmann-Arrhenius-type factor, the total probability $n_1(t)+n_2(t)$ that $x(t)<x_2^u$ is expected and numerically observed to actually exhibit a practically perfect exponential decay $e^{-\bar{\Gamma}t}$ for all $t>0$. The analytical results for $\bar{\Gamma}$ are depicted in Fig. 2(c). Their agreement with our numerical findings for the decay rate (not shown) is comparable to Fig. 1(c).

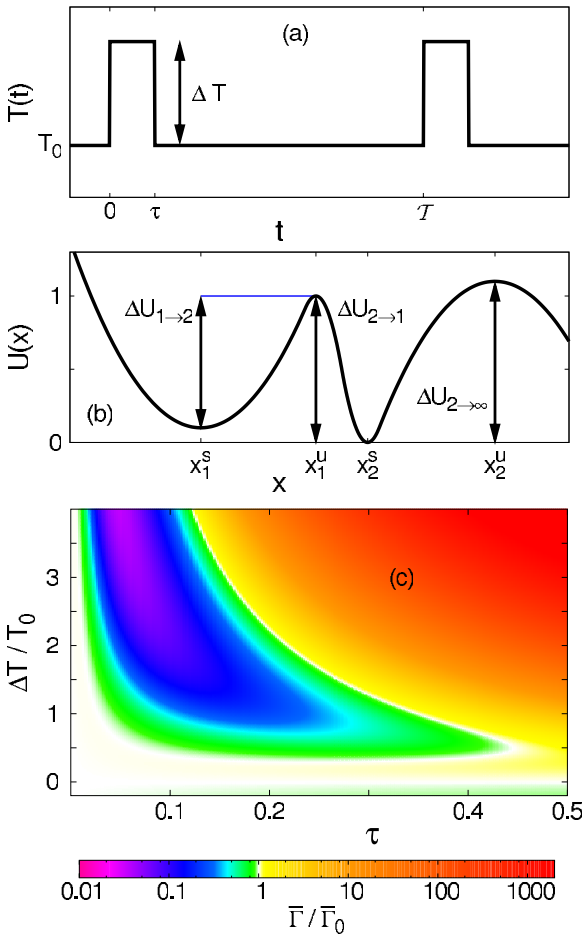


FIG. 2. (Color online) (a) Periodic temperature pulses with parameters $T_0$, $\Delta T$, $\tau$, and $\mathcal{T}$. (b) Piecewise parabolic potential with wells at $x_1^s=0$, $x_2^s=2.04$, barriers at $x_1^u=1.41$, $x_2^u=3.6$, curvatures $U''(x_1^s)=1$, $U''(x_1^u)=-10$, $U''(x_2^s)=10$, $U''(x_2^u)=-1$, and barrier heights $\Delta U_{1\to 2}=0.9$, $\Delta U_{2\to 1}=1$, $\Delta U_{2\to\infty}=1.1$. (c) Dependence of the effective rate $\bar{\Gamma}$ across $x_2^u$ on $\tau$ and $\Delta T/T_0$ in units of the rate $\bar{\Gamma}_0=9.11\times 10^{-11}$ for $T(t)\equiv T_0$. Shown are analytical results for $\mathcal{T}=30$, $kT_0=0.05$, and $\eta(t)\equiv 1$ obtained as detailed in the main text.

The most striking feature of Fig. 2(c) is a substantial *reduction* in the net escape rate $\bar{\Gamma}$ upon superimposing temperature pulses of suitable duration $\tau$ and amplitude $\Delta T$ to the "unperturbed" temperature $T_0$. Roughly speaking, $\Gamma(t)$ in Fig. 1(b) approaches the instantaneous Kramers rate the quicker, the larger the curvatures in Fig. 1(b) are [14]. Since the curvatures relevant for $\bar{\Gamma}_{2\to 1}$ are larger than those for $\bar{\Gamma}_{1\to 2}$ and $\bar{\Gamma}_{2\to\infty}$ [see Fig. 2(b)], sufficiently small $\tau$ mainly affect $\bar{\Gamma}_{2\to 1}$ and thus lead to a reduction in the net decay rate $\bar{\Gamma}$. We verified that already a single temperature pulse [Fig. 1(a)] indeed yields an analogous reduction in escapes events. Figure 2(c) further shows that the effect is overruled by competing secondary effects when $\tau$ and/or $\Delta T$ become too small. Finally, we have obtained very similar results also for $U(x_1^s)<U(x_2^s)$, but from the viewpoint of equilibrium rates [1], the case $U(x_1^s)>U(x_2^s)$ shown in Fig. 2 seems even more surprising to us.

Experimentally, potentials like in Fig. 2(b) are ubiquitous in the context of chemical reactions. For example, in the modified case $U(x_1^s) < U(x_2^s)$ these are reactions proceeding in two steps via an intermediate (metastable state $x_2^s$). Temperature pulses could be generated, among others [7], by means of a flashing black body radiator. More realistic are short laser pulses [17], whose basic effects (on the reacting molecules *and* their environment) may still be roughly modeled by a temperature pulse. On the other hand, we expect that our above-mentioned main finding will be qualitatively robust against various modifications of the pulsed perturbation, including more realistic models for tailored laser pulses. A further experimental playground are colloidal particles in a suitably designed potential landscape by exploiting light [18], dielectrophoretic [19], or magnetic [20] forces. Temperature pulses in the form of acoustomechanical white noise may be generated by means of piezoelements [21].

Since temperature pulses in Eq. (1) are basically equivalent to potential modulations in Eq. (2), we arrive at yet another quite astonishing conclusion: the escape rate for a $t$-independent temperature and a potential landscape as in Fig. 2(b) may *decrease* if the amplitude (multiplicative factor) of the potential is temporally *reduced* without any other change in its "shape." This effect should be readily observable with colloidal systems [18–20] and possibly also with cold atoms in laser-induced optical lattices [22] or in complex reaction networks [23].

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